Appendix A: Sample Calculations and Models

Nomenclature

A, Aspiration ratio
A_i, Inlet area (Expansion/Contraction), m^2
A_o, Outlet area (Expansion/Contraction), m^2
C, Cunningham correction factor
c, Concentration of particles at outlet
c_o, Concentration of particles at inlet
D_o, Outlet diameter (Expansion/Contraction), m
D_p, Particle diameter, m
D_{pr}, Probe diameter, m
D_{sh}, Shroud diameter, m
D_t, Tube diameter, m
f, Friction factor
Fr, Froude number
I, Impaction efficiency
K, Transport coefficient
k_B, Boltzmann constant
L, Transport system length, m
P, Penetration, %
Pl, Particle lift number
Q, Flow rate, lpm
R_0, Curvature ratio
Re, Reynolds number
Stk, Stokes number
T, Transmission ratio
U_i, Inlet velocity, m/s
U_{mean}, Mean velocity, m/s
U_0, Free Stream Velocity, m/s
V_B, Velocity due to Brownian motion, m/s
V_D, Deposition velocity, m/s
V_g, Gravitational settling velocity, m/s
Wl, Wall loss

Greek Symbols

φ, Inclination of transport system with the horizontal plane
μ_{air}, Dynamic viscosity of air, N.s/m
ν_{air}, Kinematic viscosity of air, m^2/s
$\theta_{1/2}$, Half angle (Expansion/Contraction), degrees
θ_b, Bend angle, degrees
ρ_{air}, Air density, kg/m^3
ρ_p, Particle density, kg/m^3
ρ_w, Density of water, kg/m^3
τ, Gravitational settling time, sec
τ_d, Aerosol particle deflection time constant, sec

A.1.1 Laminar Flow in Tubes

- **Particle Deposition in Aerosol Sampling Lines Caused by Turbulent Diffusion and Gravitational settling, N.K Anand and A.R.McFarland,**
  ***Am. Ind. Hyg. Assoc. J .50(6)*: 307-312(1989)**
- **Deposition of Particle in turbulent flow on channel or pipe walls, S.K Beal**
  ***Nuclear Science and Engineering*: 40, I-11 (1970)**

- **Data:**

D_t = 25.4 mm
L = 2 m
φ = 0 deg
Q = 20 lpm

- **Results from use of DEPOSITION 2001:**

  P = 63.3%
  Stk = 0.0078
  Re = 1066

- **Results from hand calculations:**

$$U_{mean} = \frac{4Q}{\pi D_t^2} = 0.6578 \text{m/s}$$

$$\text{Re} = \frac{\rho_{air} U_{mean} D_t}{\mu_{air}} = 1065.7$$

$$f = \frac{0.3164}{4\,\text{Re}^{0.25}} = 0.01384$$

$$Stk = \frac{C\rho_p D_p^2 U_{mean}}{9\mu_{air} D_t} = 0.0158$$

$$V_g = \tau g Sin\phi = \frac{gC\rho_w D_p^2}{18\mu_{air}} = 2.946 * 10^{-3} \text{ m/s}$$

$$V_B = \sqrt{\frac{K_B T}{2\pi m}} = 3.506 * 10^{-5} \text{ m/s}$$

$$D_p^+ = \frac{D_p U_{mean} \sqrt{f/2}}{\nu_{air}} = 0.0349$$

$$\nu_f^+ (D_p^+ / 2) = 0.05 D_p^+ / 2 = 8.726 * 10^{-4}$$

$$S^+ = \frac{U_{mean}\sqrt{f/2}}{\nu_{air}}\left[\frac{0.05U_{mean}D_p^2\rho_p\sqrt{f/2}}{\mu_{air}} + \frac{D_p}{2}\right] = 0.0691$$

$$\nu_f^+(S^+) = 0.05S^+ = 3.453*10^{-3}$$

$$V_f = \frac{U_{mean}\sqrt{f/2}}{4}\left[\nu_f^+(D_p^+/2) + \nu_f^+(S^+)\right] = 5.9181*10^{-5}\,\text{m/s}$$

$$\nu = V_f + V_B = 9.4241*10^{-5}\,\text{m/s}$$

$$V_D = \frac{1}{\frac{1}{\nu} + \frac{1}{K}} = 9.4240*10^{-5}\,\text{m/s}$$

$$\theta = \tan^{-1}\left[\frac{V_D^+}{\sqrt{|V_g^{+2} - V_D^{+2}|}}\right] = 0.03147\text{rad}$$

$$V_e = (0.5 + \frac{\theta}{\pi})V_D + \frac{V_g\cos\theta}{\pi} = 0.001002\text{m/s}$$

$$P = \frac{c}{c_0} = \exp\left[\frac{-\pi D_t V_e L}{Q}\right] = 63.3\%$$

A.1.2 Turbulent Flow in Tubes

- **Particle Deposition in Aerosol Sampling Lines Caused by Turbulent Diffusion and Gravitational settling, N.K Anand and A.R.McFarland,**
  ***Am. Ind. Hyg. Assoc. J .50(6)*: 307-312(1989)**
- **Deposition of Particle in turbulent flow on channel or pipe walls, S.K Beal**
  ***Nuclear Science and Engineering*: 40, I-11 (1970)**

- **Data:**

D_t = 25.4 mm
L = 2 m
φ = 0 deg
Q = 100 lpm

- **Results from use of DEPOSITION 2001:**

  P = 90.7 %
  Stk = 0.0389
  Re = 5331

- **Results from hand calculations:**

$$U_{mean} = \frac{4Q}{\pi D_t^2} = 3.289 \text{m/s}$$

$$\text{Re} = \frac{\rho_{air} U_{mean} D_t}{\mu_{air}} = 5329$$

$$f = \frac{0.3164}{4\,\text{Re}^{0.25}} = 0.00926$$

$$Stk = \frac{C\rho_p D_p^2 U_{mean}}{9\mu_{air} D_t} = 0.079$$

$$V_g = \tau g Sin\phi = \frac{gc\rho_w D_p^2}{18\mu_{air}} = 2.993*10^{-3} \text{m/s}$$

$$V_B = \sqrt{\frac{K_B T}{2\pi m}} = 3.506*10^{-5} \text{m/s}$$

$$D_p^+ = \frac{D_p U_{mean} \sqrt{f/2}}{\nu_{air}} = 0.1427$$

$$\nu_f^+ (D_p^+ / 2) = 0.05 D_p^+ / 2 = 3.568*10^{-3}$$

$$S^{+} = \frac{U_{mean}\sqrt{f/2}}{\nu_{air}}\left[\frac{0.05U_{mean}D_p^2\rho_p\sqrt{f/2}}{\mu_{air}} + \frac{D_p}{2}\right] = 0.9348$$

$$\nu_f^{+}(S^{+}) = 0.05S^{+} = 0.04764$$

$$V_f = \frac{U_{mean}\sqrt{f/2}}{4}\left[\nu_f^{+}(D_p^{+}/2) + \nu_f^{+}(S^{+})\right] = 2.8147*10^{-3}\,\text{m/s}$$

$$\nu = V_f + V_B = 2.85*10^{-3}\,\text{m/s}$$

$$V_D = \frac{1}{\frac{1}{\nu} + \frac{1}{K}} = 2.8499*10^{-3}\,\text{m/s}$$

$$\theta = \tan^{-1}\left[\frac{V_D^{+}}{\sqrt{|V_g^{+2} - V_D^{+2}|}}\right] = 1.258\text{rad}$$

$$V_e = (0.5 + \frac{\theta}{\pi})V_d + \frac{V_g Cos\theta}{\pi} = 0.002861\text{m/s}$$

$$P = \frac{c}{c_0} = \exp\left[\frac{-\pi D_t V_e L}{Q}\right] = 90.7\%$$

A.2.1 Laminar flow in bends

- **Motion of Particles in Bends of Circular Pipes.**
  Y.S Cheng and C.S. Wang
  ***Atmospheric Environment.* Vol. 15, pp, 301-306**

- **Data:**

D_t = 25.4 mm
R_0 = 4.0
θ_b = 60.0 deg (clockwise)
Q= 20 lpm

- **Results from use of DEPOSITION 2001:**

  P = 98.5 %
  Stk = 0.0078
  Re = 1066

- **Results from hand calculations:**

$$U_{mean} = \frac{4Q}{\pi D_t^2} = 0.6578\text{m/s}$$

$$Stk = \frac{C\rho_p D_p^2 U_{mean}}{9\mu_{air} D_t} = 0.0158$$

$$K_\alpha = \frac{4Stk}{R_0} = 0.0158$$

$$A = \left[\frac{1 + (1 + K_\alpha{}^2)^{1/2}}{2}\right]^{1/2} = 1.000031$$

$$B = \frac{K_\alpha}{2A} = 0.0079$$

$$\xi(\tau) = \cos B\tau\left[\cosh A\tau + \left(\frac{A^3}{A^2 + B^2}\right)\sinh A\tau\right] - \left(\frac{B^3}{A^2 + B^2}\right)\sin B\tau \cosh A\tau$$

$$\eta(\tau) = \sin B\tau\left[\sinh A\tau + \left(\frac{A^3}{A^2 + B^2}\right)\cosh A\tau\right] + \left(\frac{B^3}{A^2 + B^2}\right)\cos B\tau \sinh A\tau$$

$$\tan\theta_b = \frac{\eta(\tau_\theta)}{\xi(\tau_\theta)} = 1.732$$

$$\tau_\theta = 132.3637$$

$$z_\theta = \left(1 - \frac{R_0^2[\eta(\tau_\theta) - e^{\tau_\theta}\sin\theta_b]^2}{[\eta(\tau_\theta) + e^{\tau_\theta}\sin\theta_b]^2}\right) = 0.999966$$

$$I = 1 - \frac{1}{\pi R_0}\left(\left[\frac{e^{2\tau_\theta}\sin^2\theta_b}{\eta^2(\tau_\theta)} - 1\right]\left[(R_0^2 + 1)z_\theta - \frac{z_\theta^3}{3}\right] + R_0\left[\frac{e^{2\tau_\theta}\sin^2\theta_b}{\eta^2(\tau_\theta)} + 1\right](z_\theta\sqrt{1 - z_\theta^2} + \sin^{-1} z_\theta)\right)$$

If Stk < 0.02

$$I = \left(\frac{2}{\pi} + \frac{1}{R_0} + \frac{4}{3\pi R_0^2}\right) Stk\theta_b = 0.01538$$

$$P = 98.5\%$$

A.2.2 Turbulent flow in bends

- **Aerosol Deposition in bends with Turbulent Flow**
  A.R.McFarland, H.Gong, A.Muyshondt, W.B.Wente and N.K.Anand.
  ***Environ. Sci. Technol.* 31, 3371-3377 (1997)**

- **Data:**

D_t = 25.4 mm
R_0 = 4.0
θ = 90.0 deg (clockwise)
Q= 100 lpm

- **Results from use of DEPOSITION 2001:**

  P = 99.6 %
  Stk = 0.0389
  Re = 5331

- **Results from hand calculations:**

$$U_{mean} = \frac{4Q}{\pi D_t^2} = 3.289\text{m/s}$$

$$Stk = \frac{C\rho_p D_p^2 U_{mean}}{9\mu_{air} D_t} = 0.0789$$

$$a = -0.9526 - 0.05686\delta = -1.18004$$

$$b = \frac{-0.297 - 0.0174\delta}{1 - 0.07\delta + 0.0171\delta^2} = -0.3690$$

$$c = -0.306 + \frac{1.895}{\sqrt{\delta}} - \frac{2}{\delta} = 0.1415$$

$$d = \frac{0.131 - 0.0132\delta + 0.000383\delta^2}{1 - 0.129\delta + 0.0136\delta^2} = 0.1202$$

$$\ln(P) = \frac{4.61 + a\theta Stk}{1 + b\theta s Stk + c\theta Stk^2 + d\theta^2 Stk} = 4.5593$$

$$P = 99.6\%$$

A.3.1 Turbulent flow in Unshrouded Isokinetic Probes

- **On the Aspiration Characteristics Of large Diameter Thin-Walled Aerosol Sampling Probes At Yaw Orientations With Respect to the Wind**
  J.H. Vincent, D.C, Stevens, D.Mark, M. Marshall and T.A.Smith
  ***J. Aerosol. Sci.* Vol. 17, No.2, pp.211-224, (1996)**
- **Aerosol Deposition in Sampling Probes**
  B.J.Fan, F.S. Wong, A.R.McFarland, N.K. Anand
  ***Aerosol Science and Technology* 17:326-332(1992)**

- **Data:**

D_t = 25.4 mm
Q = 180 lpm
U_0 = 9m/s

- **Results from use of DEPOSITION 2001:**

  P = 92.3
  Stk = 0.0700
  Re = 9597
- **Results from hand calculations:**

$$D_{pr} = \sqrt{Q/\pi U_{mean}} = 20.6\text{mm}$$

$$A = 1$$

$$\mathrm{Re}_{pr} = \frac{D_{pr} U_{mean}}{\nu} = 11826$$

$$Stk = \frac{2\tau U_{mean}}{D_{pr}} = 0.2012$$

$$Pl = \frac{2\tau_d U_{mean}}{D_{pr}} = 10.27$$

$$Fr = \frac{2U_{mean}^2}{gD_{pr}} = 18.39$$

$$Wl = 1.769\left(1 + \frac{L}{Fr}\right)^{-9.19} R_s^{0.559} R_e^{-0216} = 0.0781$$

$$P = A(1 - Wl) = 92.19\%$$

A.3.2 Laminar flow in Unshrouded Anisokinetic Probes

- **Data:**

D_t = 25.4 mm
D_{pr}= 18.2 mm
Q = 20 lpm
U_0 = 6m/s

- **Results from use of DEPOSITION 2001:**

P = 164.2 %
Stk = 0.0078
Re = 1066

- **Results from hand calculations:**

$$U_{mean} = \frac{4Q}{\pi D_{pr}^2} = 1.2813\text{m/s}$$

$$R = U_0 / U_{mean} = 4.683$$

$$A = 1 + \left[1 - \frac{1}{1 + 1.05Stk}\right](R - 1) = 1.6424$$

$$\text{Re}_{pr} = \frac{D_{pr}U_{mean}}{\nu} = 1487$$

$$Stk = \frac{2\tau U_{mean}}{D_{pr}} = 0.043$$

$$Pl = \frac{2\tau_d U_{mean}}{D_{pr}} = 4.1219$$

$$Fr = \frac{2U_{mean}^2}{gD_{pr}} = 18.39$$

$$Wl = 1.769\left(1 + \frac{L}{Fr}\right)^{-9.19} R_s^{0.559} R_e^{-0216} = 0.000842$$

$$P = A(1 - Wl) = 164.1\%$$

A.3.3 Turbulent flow in Shrouded Probes

- **A Predictive Model for Aerosol Transmission through a Shrouded Probe. Hongrui Gong, Sumit Chandra, Andrew McFarland, and N.K. Anand *Environ. Sci. Technol*. 30, 3192-3198 (1996).**

- **Data:**

U_0 = 3.0 m/s
D_t = 25.4 mm
D_{pr} = 18.2 mm
D_{sh} = 40.0 mm
VRR = 1.25 (velocity reduction ratio)
Q =100 L/min

- **Results from use of DEPOSITION 2001:**

P = 83.7 %
Stk = 0.0389
Re = 5331

- **Results from hand calculations:**

$$U_s = \frac{4Q}{\pi D_{sh}^2} = 0.77\text{m/s}$$

$$Stk_{sh} = \frac{C\rho_p D_p^2 U_0}{9\mu_{air} D_{sh}} = 0.0349$$

$$R_{sh} = \frac{U_0}{U_{sh}} = 3.896$$

$$F = 1-(R_{sh}-1)\frac{0.861 Stk_{sh}}{[2.34+0.939(R_{sh}-1)]Stk_{sh}+1} = 0.926$$

$$\text{Re} = \frac{\rho U_0 D_t}{\mu} = 1066$$

$$A_{sh} = 1+\frac{1.05 Stk_{sh}}{1+1.05 Stk_{sh}}\left(R_{sh}-1\right) = 1.102$$

$$U_{sc} = U_{sh}\left[1+1.45\left(1-\frac{1+\ln R_{sh}}{R_{sh}}\right)\right] = 1.210\text{m/s}$$

$$R_{pr} = \frac{U_{sc}}{U_{pr}} = 0.1888$$

$$A_{pr} = 1+\frac{1.05 Stk_{pr}}{1+1.05 Stk_{pr}}\left(R_{pr}-1\right) = 0.851$$

$$Wl = 0.496\left(1+\frac{L}{Fr}\right)^{0.194} Stk_{pr}^{\ 0.613}\left(\frac{U_{sh}}{U_{pr}}\right)^{1.191} \approx 0.0155$$

$$T = FA_{sh}A_{pr}\left(1-Wl\right) = 83.7\%$$

A.4.1 Turbulent flow in Contractions

- **Deposition of Aerosol Particles in Contraction Fittings**
  Arnold Muyshondt, A.R.McFarland and N.K.Anand.
  ***Aerosol Science and Technology* 24:205-216 (1996).**

- **Data:**

  $D_i = 25.4$ mm
  $\theta_{1/2} = 45.0$ deg
  $D_e = 22.86$ mm
  $Q = 100$ L/min

- **Results from use of DEPOSITION 2001:**

  $P = 99.6$ %
  $Stk = 0.0389$
  $Re = 5331$

- **Results from hand calculations:**

$$Stk = \frac{C\rho_p D_p^2 U_i}{9\mu D_e} = 0.0497$$

$$X = Stk\left(1 - \frac{A_e}{A_i}\right) = 0.00945$$

$$Wl = \frac{1}{1 + \left[\frac{3.14 e^{-0.01849\theta_{1/2}}}{X}\right]^{1.244}} = 0.0021$$

$$P = 1 - Wl = 99.8\%$$

A.4.2 Turbulent flow in Expansions

- **Aerosol Deposition in Transport Lines,**
  Arnoldo Muyshondt
  ***Ph.D Dissertation,* 1995**

- **Data:**

D_t = 25.4 mm
$\theta_{1/2}$ = 45.0 deg
D_o = 30.0 mm
Q = 100 lpm

- **Results from use of DEPOSITION 2001:**
  P = 98.4%
  Stk = 0.0389
  Re = 4514

- **Results from hand calculations:**

$$Stk = \frac{C\rho_p D_p^2 U_{outlet}}{9\mu_{air} D_{inlet}} = 0.0567$$

$$R = 1 - \frac{A_i}{A_o} = 0.2832$$

$$Wl = 1.1358R^2 e^{\left[0.5\left[\left[\frac{\ln\left(\frac{Stk_c R}{0.5518}\right)}{1.9661}\right]^2 + \left[\frac{\ln\left(\frac{\theta_{1/2}}{12.519}\right)}{2.7825}\right]^2\right]\right]} = 0.016$$

$$P = 1 - Wl = 98.4\%$$

A.5.1 Turbulent flow in Commercial Probes (RF-2-111)

- **Models: Curve fits from experimental data.**

- **Data:**

D_{pr} = 25.4 mm (diameter of inner probe)
Q = 57 L/min
U_0 = 10 m/s

- **Results from use of DEPOSITION 2001:**

P = 92.1 %
Stk = 0.0222
Re = 3039

- **Results from hand calculations:**

If: 2.5m/s < U_0 < 24m/s

$$P = 1.000 - 0.034261U_0 + 0.00237U_0^2 + 5.73*10^{-5}U_0^3 - 3.05*10^{-6}U_0^4 = 92.1\%$$

A.5.2 Turbulent flow in Commercial Probes (RF-2-112)

- **Results from use of DEPOSITION 2001:**

P = 117.8 %
Stk = 0.0389
Re = 5331

- **Results from hand calculations:**

$$P = \frac{1}{\left(1.0527929 - 0.0064409386U_0^{1.5} - 0.052824346e^{-U_0}\right)} = 117.8\%$$

If 0.5m/s < U_0 < 15m/s

A.5.3 Turbulent flow in Commercial Probes (RF-2-113)

- **Results from use of DEPOSITION 2001:**

P = 120.8%
Stk = 0.0389
Re = 5331

- **Results from hand calculations:**

If 0.5m/s < U_0 < 10m/s

$$P = 0.84500461 + 0.036283175U_0 + 0.15327539e^{-U_0} = 120.78\%$$

A.5.4 Turbulent flow in Commercial Probes (CMR4CFM HI)

- **Results from use of DEPOSITION 2001:**

  P = 99.9 %
  Stk = 0.0389
  Re = 5331

- **Results from hand calculations:**

If 10m/s < U_0 < 60 m/s

$$P = 0.9892624 + 0.00032130494U_0^{1.5} - 6.5613425 * 10^{-16} e^{U_0} = 99.94\%$$

A.5.5 Turbulent flow in Commercial Probes (CMR4CFM MI)

- **Results from use of DEPOSITION 2001:**

  P = 100.3%
  Stk = 0.0389
  Re = 5331

- **Results from hand calculations:**

If 1m/s < U_0 < 25 m/s

$$P = 0.99941742 + 0.0049074179U_0^2 - 0.0008766672U_0^{2.5} - 0.066421873U_0^{0.5} = 100.3\%$$

A.5.6 Turbulent flow in Commercial Probes (WW6-111)

- **Results from use of DEPOSITION 2001:**
  P = 107.7%
  Stk = 0.0389
  Re = 5331

- **Results from hand calculations:**

If 1m/s < U_0 < 20m/s

$$P = \frac{1}{1.1136111 - 0.025451089U_0^{0.5} \log U_0} = 107.7\%$$

A.5.7 Turbulent flow in Commercial Probes (KCI-III)

- **Results from use of DEPOSITION 2001:**
  P =110.1%
  Stk = 0.0389
  Re = 5331

- **Results from hand calculations:**

If 0.5m/s < U_0 < 20m/s

$$P = 0.88571429 + 0.0021510204U_0^2 = 110.1\%$$

A.6.1 Turbulent Flow in Splitters

- **Experimental Study Of Aerosol Deposition in Flow Splitters with Turbulent Flow**
  Rajiv Gupta and A.R. McFarland
  AST Manuscript, 99071

- **Data:**
  D_t = 25.4 mm
  Q = 100 lpm
  D_o = 17.78 mm
  θ = 30 deg
- **Results from use of DEPOSITION 2001:**

  P = 96.6%
  Stk = 0.0389
  Re = 5331

- **Results from hand calculations:**

If SplitterDia/Tube dia = 0.7 and 0.034<Stk<1.25 and 2560 < Re <13600,

$$\ln P = -2.635431 + \frac{2.623013}{1+\left(\frac{\text{Stk}}{0.457332}\right)^{1.679914}} + \frac{2.291426}{1+\left(\frac{\theta}{56.451397}\right)^{3.869940}} - \frac{2.287903}{\left[1+\left(\frac{\text{Stk}}{0.457332}\right)^{1.679914}\right]\left[1+\left(\frac{\theta}{56.451397}\right)^{3.86994}\right]} = -0.03497$$

$P = 96.56\%$

Note: There is no difference in the penetration in the left and right branches.

A.6.2 Laminar Flow in Splitters

- **Experimental Study Of Aerosol Deposition in Flow Splitters with Turbulent Flow**
  Rajiv Gupta and A.R. McFarland
  AST Manuscript, 99071

  Data:
  D_t = 25.4 mm
  Q = 20 lpm
  D_o = 17.78 mm
  θ = 30 deg

- **Results from use of DEPOSITION 2001:**
  P =99.3%
  Stk = 0.0158
  Re = 1066

- **Results from hand calculations:**

$$Stk = \frac{C_a \rho_p D_p^2 U_t}{9 \mu_{air} D_t} = 0.0158$$

$$P = \frac{1}{1 + \left(\frac{Stk}{1.789} \right)^{1.051}} = 99.3\%$$